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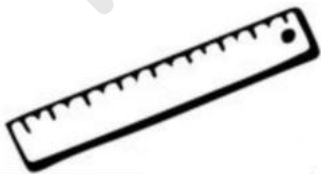
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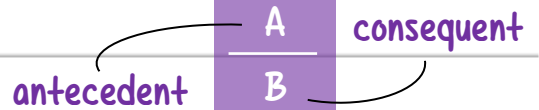


chapter 1

Ratio & Proportion

Ratio

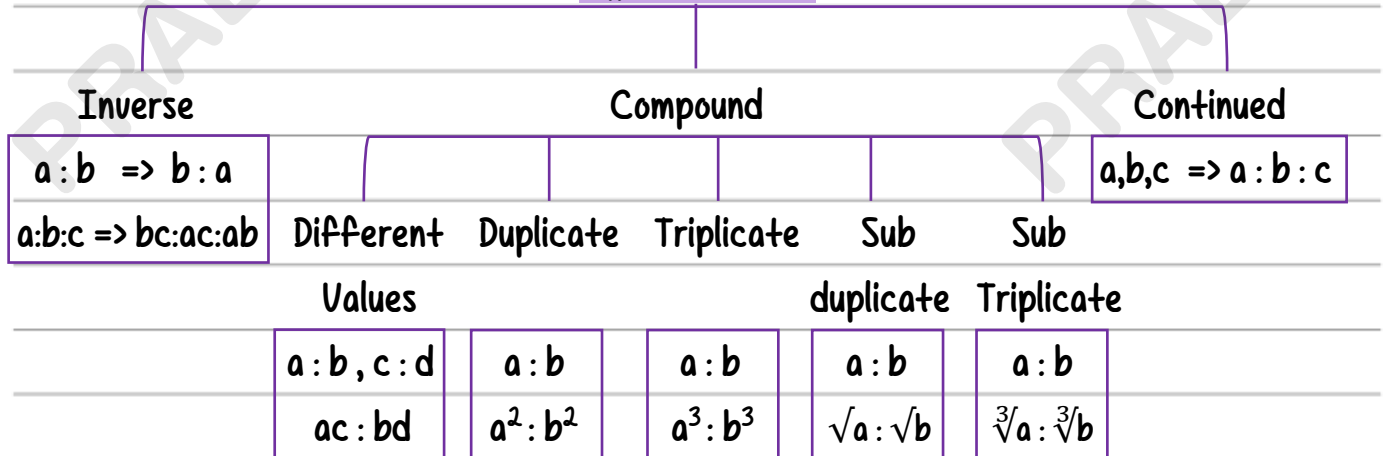
comparison of ≥ 2 quantities of **same kind** by division



NOTES:

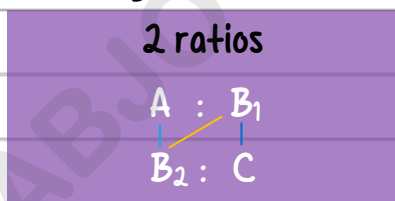
- Order of the terms is important and cannot be **changed**
- Ratios can be multiplied/divided by a common number but **not** added/subtracted
- If number is \uparrow/\downarrow in ratio $x:y$, then **[New quantity = $y/x \times$ old number]**

Types of Ratios

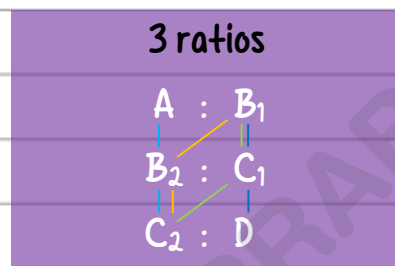


Exam tips

- ✓ If 3,000 is to be allocated in $a : b \Rightarrow a = 3,000 \times \frac{a}{(a+b)}$ $b = 3,000 \times \frac{b}{(a+b)}$
- ✓ If $x:y = 3:4$, value of any equation can be obtained by taking $x=3$ and $y=4$ in equation
- ✓ If ratios are given to arrive at continued ratio:



$AB_2 : B_2B_1 : B_1C$



$AB_2C_2 : C_2B_2B_1 : C_2C_1B_1 : B_1C_1D$

<h1>Proportion</h1>	Equality of 2 ratios	$A : B :: C : D$ <p>(means) (extremes)</p>	$\frac{A}{B} = \frac{C}{D}$ $AD = BC$
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NOTES:

○ a,b,c,d are called 1st, 2nd, 3rd and 4th proportion respectively

○ Continued proportion: a,b,c => a : b :: b : d.

Note: Continued proportion are always in Geometric progression (GP)

○ Mean proportional: In continued proportion, 'b' is mean proportional. i.e., $[b^2 = ac]$

Assume: $\frac{A}{B} = \frac{C}{D}$		Properties of Proportions					Theory
Invertendo	Alternendo	Componendo	Dividendo	Comp & Divi	Addendo	Subtrahendo	
$\frac{B}{A} = \frac{D}{C}$	$\frac{A}{C} = \frac{B}{D}$	$\frac{A+B}{B} = \frac{C+D}{D}$	$\frac{A-B}{B} = \frac{C-D}{D}$	$\frac{A+B}{A-B} = \frac{C+D}{C-D}$	$\frac{A+C}{B+D}$	$\frac{A-C}{B-D}$	
A = C	C = D	B D	B D	A - B = C - D	B + D	B - D	

Exam tips

✓ If $x = y = z$, value of any equation can be obtained by taking $x=3, y=4$ & $z = 5$ in equation

3 4 5

chapter 1

Indices & logarithms

Indices

representation of numbers in terms of **powers** to small base

base a^n power

Laws of indices:

- $a^m \times a^n = a^{m+n}$
- $a^m \div a^n = a^{m-n}$
- $(a^m)^n = a^{m \times n}$
- $a^0 = 1$
- $\sqrt[n]{a} = a^{1/n} = a^{-n}$
- If $a^x = a^y$, then $x = y$
- If $a = b$, then $a^b = b^a$
- $\sqrt{a \sqrt{a \sqrt{a \dots \infty}}} = a$
- $\sqrt{a \sqrt{a \sqrt{a \dots n \text{ times}}}} = a^{\frac{2^n - 1}{2^n}}$
- If $x = a^{1/2} + a^{-1/2}$, then $x^2 - 2 = a + 1/a$
- If $x = a^{1/2} - a^{-1/2}$, then $x^2 + 2 = a + 1/a$
- If $x = a^{1/3} - a^{-1/3}$, then $x^3 + 3x = a - 1/a$
- If $x = a^{1/3} + a^{-1/3}$, then $x^3 - 3x = a + 1/a$

Algebraic equations:

- $(a + b)^2 = a^2 + 2ab + b^2$
- $(a - b)^2 = a^2 - 2ab + b^2$
- $(a + b)(a - b) = a^2 - b^2$
- $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$
- $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$
- $a^3 + b^3 = (a + b)^3 - 3ab(a + b)$
- $a^3 - b^3 = (a - b)^3 + 3ab(a - b)$
- $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$
- $a^3 + b^3 + c^3 = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ac) + 3abc$
- If $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$

Exam tips

- ✓ Questions with complex 'n' powers can be simply solved by **substituting** 'n' with 1 or 0
- ✓ If powers are given in **cyclic** order, the resultant answer will always be a^0 i.e., 1
- ✓ If $a^x = b^y = c^z$, any equation containing x, y, z can be solved by equating base to 'k'. i.e., $a = k^{1/x}$, $b = k^{1/y}$, $c = k^{1/z}$ and solving for a x b = c

logarithms

power to which base must be raised to produce the number

$$a^n = x$$

$$\log_a x = n$$

NOTES:

- If base to the log is not mentioned, assume the same as **10** [common log]
- In a common log, 'n' can be derived as logarithm table as follow:

$$\text{Log}_{10} 4594 = \underbrace{3}_{\substack{\text{Characteristic} \\ \downarrow \\ \text{Power required to base} \\ \text{10 to reach 'n' value}}} + \underbrace{0.6623}_{\substack{\text{Mantissa} \\ \downarrow \\ \text{Derived from} \\ \text{log table}}}$$

Laws of logarithms:

- $\log_a 1 = 0$
- $\log_a a = 1$
- $\log_a m^n = n \log_a m$
- $\log_{a^b} x = 1/b \log_a x$
- $\log_a mn = \log_a m + \log_a n$
- $\log_a m/n = \log_a m - \log_a n$
- $\log_a x = 1 \div \log_x a$
- $\log_a m = \log_b m \div \log_b a$
- $a^{\log_a x} = x$
- If $\log x = \log y$, then $x = y$
- If $\log_a x = n$, then $x = \text{antilog } n$

Digits	Power
4	3
3	2
1	0
1 decimal	$\bar{1}$
2 decimals	$\bar{2}$

Exam tips

- ✓ In case of multiple logs, i.e., $\log(\log(\log m))$, compute value from innermost
- ✓ Value of $\log a$ on calculator $\Rightarrow \sqrt{a}$ (13 times) $\rightarrow -1 \rightarrow$ multiply 3558
- ✓ Value of a^n on calculator
 - If n is whole number \Rightarrow Type $a \rightarrow$ Press $x \rightarrow$ Press $=$ (n times)
 - If n has decimals $\Rightarrow \sqrt{a}$ (12 times) $\rightarrow -1 \rightarrow$ multiply $n \rightarrow +1 \rightarrow$ Press $x =$ (12 times)
- ✓ Value of $a^{1/n}$ on calculator $\Rightarrow \sqrt{a}$ (12 times) $\rightarrow -1 \rightarrow \div n \rightarrow +1 \rightarrow$ Press $x =$ (12 times)

Equations

Equations

mathematical statement of **equality**. The derivation (solution) of the unknown in the equation is known as **roots**.

Types of Equations:

- **Simple Equation** → $ax + b = 0$, where 'x' is unknown
- **Linear Equation** → $ax + by + c = 0$, where 'x' and 'y' are unknown
- **Quadratic Equation** → $ax^2 + bx + c = 0$. "x" in this case would have upto 2 roots
- **Cubic Equation** → $ax^3 + bx^2 + cx + d = 0$, "x" in this case would have upto 3 roots

Solving Linear Equation

Two equations " $a_1x + b_1y + c_1 = 0$ " & " $a_2x + b_2y + c_2 = 0$ " can be solved by any of the ways:

Elimination method

- Step 1: Multiply Eq 1 with a_2
- Step 2: Multiply Eq 2 with a_1
- Step 3: Step 1 – Step 2 to arrive at "y"
- Step 4: Solve for "x" by substituting "y" in Eq 1 or Eq 2

Cross multiplication method

$$\begin{array}{ccc}
 \underline{x} & \underline{y} & \underline{1} \\
 b_1 & c_1 & a_1 \\
 b_2 & c_2 & a_2
 \end{array}
 \begin{array}{l}
 \nearrow \searrow \\
 \nearrow \searrow \\
 \nearrow \searrow
 \end{array}
 \begin{array}{ccc}
 a_1 & b_1 & \\
 a_2 & b_2 & \\
 & &
 \end{array}$$

i.e., $\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$

Substitution method

Ditch everything and simply substitute options in equation to check matches!

or, $x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}$ & $y = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$

Solving Quadratic Equation

The roots ' α ' & ' β ' of equation " $ax^2 + bx + c = 0$ " can be identified by any of the ways:

Factorisation method

The LCM of [a x c] should be considered such that sum of LCM results in b simplifying to the equation $(x - \alpha)(x - \beta) = 0$

Formula method

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

NOTES:

○ Nature of Roots: In a quadratic equation “ $b^2 - 4ac$ ” is known as **Discriminant**.

<i>If Discriminant</i>	<i>Nature of Roots</i>
$= 0$	Real & Equal
Perfect Square	Real, Unequal & Rational
Not Perfect Square	Real, Unequal & Irrational*
< 0	Imaginary (Does not exist)

*Irrational roots occur in **conjugate pairs** i.e., if $\alpha = p + \sqrt{q}$, then $\beta = p - \sqrt{q}$

○ Properties of Roots:

$$[\alpha + \beta] = -b/a \qquad \alpha\beta = c/a$$

○ Constructing Quadratic equation: Equation of roots ‘ α ’ & ‘ β ’ can be formed by:

$$x^2 - [\alpha + \beta]x + \alpha\beta = 0$$

Solving Cubic Equation

The roots ‘ α ’, ‘ β ’ & ‘ γ ’ of equation “ $ax^3 + bx^2 + cx + d = 0$ ” can be identified by any of the ways:

<u>Trial & Error method</u>	<u>Properties of roots</u>
Identify first root by substituting x with values that satisfy the equation. With $(x - \alpha)$ as a factor, solve for the remaining quadratic equation to obtain β & γ	$[\alpha + \beta + \gamma] = -b/a$ $\alpha\beta\gamma = -d/a$
	Identify the option which matches the properties with equation

Algebraic equations:

- $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$
- $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$
- $\alpha^3 - \beta^3 = (\alpha - \beta)[(\alpha + \beta)^2 - \alpha\beta]$
- $\alpha - \beta = \sqrt{(\alpha + \beta)^2 - 4\alpha\beta}$

Infinite series

In case of infinite series, identify the roots such that $\beta - \alpha = 1$

○ $\sqrt{x + \sqrt{x + \sqrt{x + \dots \infty}}}$ = Higher root (β)

○ $\sqrt{x - \sqrt{x - \sqrt{x - \dots \infty}}}$ = Smaller root (α)

CHAPTER 3

Linear Inequalities

Inequalities

statements where 2 quantities are **unequal** but a **relationship exists** between them.

Eg: $ax + by + c > 0$ $ax + by + c < 0$ $ax + by + c \geq 0$ $ax + by + c \leq 0$

NOTES:

- In an inequality, the sign is very important and cannot be reversed unless both sides are multiplied by -ve sign.
- Range of a variable are expressed by [lower value, upper value]. If a circle bracket “(“ is used, it means the value is not included. However, if a square bracket “[“ is used, it means, value is included. For eg, $[-2, 3)$ means variable x falls between $-2 \leq x < 3$
- Range of $\sin x$ and $\cos x$ is $[-1, 1]$

Solving Linear inequalities in two variables

Step 1: Replace inequality with an equal sign

Step 2: Find co-ordinates of line by taking $x = 0$ to get y and $y = 0$ to get x

Step 3: Plot the co-ordinates received to form straight line in graph

Step 4: Shading in graph takes place by following rule:

Scenario

Shading side

$X >$

Right

$X <$

Left

$Y >$

Up

$Y <$

Down

Consider the same only
after making
co-efficients of variables
as + ve.

Exam tips

- ✓ In a scenario based question involving, resources, people, work, time etc., an inherent basic condition is also $x \geq 0, y \geq 0$

Mathematics of finance

Simple Interest

interest on **principal** for entire period of borrowing.

No interest is paid **on interest**

$$SI = P \times N \times R$$

Labels: *P* (principal), *N* (No. of years), *R* (Interest rate p.a.)

$$A_n = P + SI \quad \text{OR} \quad A_n = P [1 + NR]$$

Label: *A_n* (Amount (Final value))

NOTES:

- R is expressed in absolute numbers (**decimals**). Not in %
- N should always be taken in **years**. Months can be converted to years by dividing by 12
- Amounts in SI are always in Arithmetical Progression (AP). i.e., $A_1, A_2, A_3 \dots$
(+)SI (+)SI

Compound Interest

interest on **progressing principal** for a defined

period of borrowing. **Interest** is also paid **on interest**

$$CI = P [(1+r)^n - 1]$$

$$A_n = P [1+r]^n$$

$$E = (1+r)^n - 1$$

Effective rate of interest

NOTES:

- r and n should be expressed in same conversion period as interest is compounded. Eg: if interest is compounded quarterly, "r" would be taken as Interest rate p.a. / 4 and 1 year would be converted to 4 periods
- Amounts in CI are always in Geometric Progression (GP). i.e., $A_1, A_2, A_3 \dots$
 $\times(1+r)$ $\times(1+r)$

Other Formulas:

- WDU of asset = Original Cost $[1 - \text{Dep}\%]^n$
- $CI_2 - SI_2 = PR^2$
- $CI_3 - SI_3 = PR^2 [R + 3]$

Annuity

series of **constant** payments/receipts made over **constant** period of time.

Present value of Annuity received in Perpetuity (indefinitely)

If no growth

$$\frac{A}{r}$$

Annuity

If growth

$$\frac{A}{r - g}$$

Present Value & Future Value

Single payment

[Invest once]

$$PV = \frac{FV}{(1+r)^n}$$

$$FV = PV(1+r)^n$$

Annuity Regular

[Invest at end of each year]

$$PV = A \times \sum_1^n \frac{1}{(1+r)^n}$$

$$FV = \left[A \times \sum_1^{n-1} (1+r)^{n-1} \right] + A$$

Annuity Due

[Invest at start of each year]

$$PV = \left[A \times \sum_1^{n-1} \frac{1}{(1+r)^{n-1}} \right] + A$$

$$FV = A \times \sum_1^n (1+r)^n$$

Exam tips

✓ Value of $\sum_1^n (1+r)^n$ on calculator $\Rightarrow (r+1) \times 1 \rightarrow$ Press = (n times) \rightarrow Press GT

✓ Value of $\sum_1^n \frac{1}{(1+r)^n}$ on calculator $\Rightarrow (r+1) \rightarrow$ Press $\div \rightarrow$ Press = (n times) \rightarrow Press GT

Applications of Time Value of Money

○ **Sinking Fund** \rightarrow Fund created by crediting series of periodic payments in annuity which is compounded annually.

$$\text{i.e., Accumulation in Sinking Fund} = \left[A \times \sum_1^{n-1} (1+r)^{n-1} \right] + A$$

○ **Leasing** \rightarrow Decision to lease an asset or purchase the asset

Lessor point of view

(Income perspective)

Lease if: PV of Lease rent > Purchase price

Lessee point of view

(Expense perspective)

PV of Lease rent < Purchase price

o Investment decision → Decision to invest in an asset/project etc. based on

Net Present Value (NPV)

$$NPV = PV \text{ of Inflows} - PV \text{ of Outflows}$$

Scenario	Decision
If NPV is +ve	Accept the Project/Asset
If NPV is 0	Indifference point
If NPV is -ve	Reject the Project/Asset

Note: Where 2 projects are given to compare, accept the project with higher NPV

o Valuation of Bond → Price at which bond can be purchased (IP) considering the interest received p.a. (I), Redemption price (RP) and expected rate of return (r)

$$IP = \frac{I}{\sum_{t=1}^n (1+r)^t} + \frac{RP}{(1+r)^n}$$

Int. (value) as per Bond rate ROI (rate) as per investor expectation

Scenario	Decision
If IP < Actual price	Purchase the Bond
If IP = Actual price	Indifference point
If IP > Actual price	Reject the Bond

o Compound Annual Growth Rate (CAGR) → Growth over period of time of a particular element (revenue, units, users etc.)

$$CAGR = \left(\frac{\text{Current value}}{\text{Initial Value}} \right)^{1/n} - 1 \times 100$$

CHAPTER 5

Permutation & Combination

Permutation

ways of selecting things with due
due attention to **arrangement**

items available ${}^n P_r$ selection

NOTES:

- **Factorial:** multiplication of all integers from 1 to n. For eg $0! = 1$, $3! = 1 \times 2 \times 3 = 6$

Permutation Formulas

$$\begin{aligned} \circ {}^n P_r &= \frac{n!}{(n-r)!} & \circ {}^n P_n &= n! & \circ {}^n P_0 &= 1 \end{aligned}$$

Combination

ways of selecting things where
arrangement is **not important**

items available ${}^n C_r$ selection

NOTES:

- In combination there is no regard to the order of arrangement. These are mere cases of selection.

Combination Formulas

$$\begin{aligned} \circ {}^n C_r &= \frac{n!}{(n-r)! r!} & \circ {}^n C_n &= 1 & \circ {}^n C_0 &= 1 \\ \circ {}^n C_r &= {}^n C_{(n-r)} \end{aligned}$$

- **Relationship:** Combination (selection) \times Arrangement = Permutation

$${}^n C_r \times r! = {}^n P_r$$

SPECIAL SCENARIOS

Arrangement of words

- **Simple Arrangement** \rightarrow Assume formed words need not have a meaning.

Eg: Arrange "RAHUL" $\Rightarrow {}^5 P_5$

- **Rearrangement** \rightarrow Question does not require the original word

Eg: Rearrange "RAHUL" $\Rightarrow {}^5 P_5 - 1$

- **Letters come together** \rightarrow Consider letters as **one unit** and provide Permutation. Separate permutation for letters inside unit unless restricted

Eg: Arrange "RAHUL" such that Vowels come together

$$\Rightarrow \text{RHL } \boxed{\text{AU}} \quad \left(\begin{array}{c} {}^4P_4 \times {}^2P_2 \\ \text{Letter} \quad \text{Vowel} \\ \text{arrangement} \quad \text{arrangement} \end{array} \right)$$

○ Letters don't come together → If particular way is restricted consider

Total ways (-) restricted ways

Eg: Arrange "RAHUL" such that Vowels don't come together $\Rightarrow {}^5P_5 - [{}^4P_4 \times {}^2P_2]$

○ Restricted places → Conditional Permutation. Draw places and identify letters allowed in restricted place. Remaining places = (No. of letters - 1) in reducing sequence.

Total ways = Multiply all selection

Eg: Arrange "RAHUL" such that R does not appear in the last place

$$\Rightarrow \begin{array}{c} _ _ _ _ _ \\ 4 \times 3 \times 2 \times 1 \times 4 \\ (A, H, U, L) \end{array}$$

Note: Permutation only works when letters to be placed are \leq the spaces available. Where spaces available are more than letters, switch to the reverse condition. Eg: If vowels in word "DAUGHTER" need to occupy odd places, Permutation cannot be done since there are 3 vowels and 4 odd spaces. Instead, reverse condition for 5 constants to occupy 4 even spaces and 1 odd space.

Arrangement of Digits

○ Simple Arrangement → Assume Digits are **not repeating**

Eg: Form 4 Digit number using 1 to 5 $\Rightarrow {}^5P_4$

○ Single Place restriction → Conditional Permutation. Draw places and identify Digits allowed in restricted place. Remaining places = (No. of Digits - 1) in reducing sequence.

Eg: Form 4 Digit number $> 5,000$ using 3 to 7 $\Rightarrow \begin{array}{c} _ _ _ _ \\ 3 \times 4 \times 3 \times 2 \\ (5,6,7) \end{array}$

○ Multiple Place restrictions → Split restrictions. Fix primary restrictions and apply conditional permutation. Total ways = Sum of all ways

Eg: Form 4 Digit number $> 2,300$ using 1 to 5

\Rightarrow Between 2000 - 2999

(+)

\Rightarrow Above 2999

$$\begin{array}{c} 2 \quad - \quad - \quad - \\ 1 \times 3 \times 3 \times 2 \\ (3,4,5) \end{array}$$

$$\begin{array}{c} - \quad - \quad - \quad - \\ 3 \times 4 \times 3 \times 2 \\ (3,4,5) \end{array}$$

Note: If 0 is given as a Digit, it would be considered as a restriction for the first place as that would reduce the Digit formation. Consider Conditional Permutation accordingly

Eg: 4 Digit number using 0, 1, 2, 3, 4, 5 \Rightarrow

$$\begin{array}{c} - \quad - \quad - \quad - \\ 4 \times 4 \times 3 \times 2 \\ (1,2,3,4,5) \end{array}$$

Sum of Numbers formed with Digits

○ No repetitions \rightarrow Shortcut is $[\text{Sum of Digits}](n-1)! \times 1111$

$\underbrace{\hspace{1.5cm}}_{\text{No. of digits}} \quad \underbrace{\hspace{1.5cm}}_{\substack{\text{No. of 1=} \\ \text{No. of digits}}}$

Eg: Sum of all 4-digit number with 1, 3, 5, 7 $\Rightarrow [1+3+5+7] \times 3! \times 1111$

○ Digits repeating \rightarrow Divide Sum with (No. of Digits repeating)!

Eg: Sum of all 4-digit number with 1, 3, 3, 5 $\Rightarrow [1+3+3+5] \times 3! \times 1111$

2!

○ Digits contain 0 \rightarrow Since 0 cannot appear in the first place, Compute Sum of all Digits

(-) Sum of Digits if 0 occurs in first place

Eg: Sum of all 4-digit numbers with 0, 1, 3, 5

$\Rightarrow [0+1+3+5] \times 3! \times 1111$ (-) $[1+3+5] \times 2! \times 111$

Factorisation

○ Total Factors \rightarrow To find the total number of factors of a number, identify the prime factors in the number and report them in indices i.e., $(p)^a (q)^b (r)^c$

$$\text{Total Ways} = (a+1)(b+1)(c+1)$$

○ Different Factors \rightarrow Since the number is a factor of itself, too,

$$\text{Different Ways} = [(a+1)(b+1)(c+1)] - 1$$

Repetition

○ **Undefined repetitions** If same object can be repeated any number of times in the placements, Ways = n^r

Eg: Form 4 digit number using any number between 1 to 9. Same number can be used again

$$\Rightarrow \begin{array}{c} _ _ _ _ \\ 9 \times 9 \times 9 \times 9 \end{array} \quad (\text{or}) \quad 9^4$$

○ **Identified repetitions** If certain objects are repeated in a given placement, impact of repetitions would be removed by dividing the No. of ways by (No. of repetitions used)!

Eg: Arrange "AGARWAL" using all letters $\Rightarrow \frac{{}^7P_7}{3!}$ ← Repetitions of A

Note: The above only works since all objects given are used in the placement. Where placements given are less than a objects available, given objects will need to be grouped and identified for various possibilities where repetitions may occur/not occur

Eg: Form 4 letter word \Rightarrow 4 letter word can be formed by using the following unique letters: AAA G R W L

Possibilities:	No repetitions	2 letters repeating	3 letters repeating
	${}^5C_4 \times 4!$	$+ \frac{{}^3C_2 \times {}^4C_2 \times 4!}{2!}$	$+ \frac{{}^3C_3 \times {}^4C_1 \times 4!}{3!}$
	↙ Arrangement	2! ← Repetitions of A	→ 3!

Occurrence

○ **Item always occurs** \rightarrow Use Combination to fill other available places.

Arrangement of places = (Total places)!

Eg: Arrange 10 items in 4 places such that 1 item always occurs $\Rightarrow {}^9C_3 \times 4!$

○ **Item never occurs** \rightarrow Ignore the item as if it was never part of the list. Can use Permutation directly.

Eg: Arrange 10 items in 4 places such that 1 item never occurs $\Rightarrow {}^9P_4$

Eg: Split 12 students into 3 groups $\Rightarrow \frac{12!}{4! 4! 4! 3!}$

Open Selection

Different things

○ Zero or more selections \rightarrow If zero or more different things can be selected,

$$\text{Ways} = 2^n$$

○ One or more selections \rightarrow If atleast one different things needs to be selected,

$$\text{Ways} = 2^n - 1$$

○ Multiple different things \rightarrow Multiplication of all ways i.e.,

$$\text{Zero or more} = 2^{n_1} \times 2^{n_2}$$

$$\text{Atleast one} = (2^{n_1} - 1)(2^{n_2} - 1)$$

Alike things

○ Zero or more selections \rightarrow If zero or more alike things can be selected,

$$\text{Ways} = (n+1)$$

○ One or more selections \rightarrow If atleast one alike things needs to be selected,

$$\text{Ways} = n$$

○ Multiple alike things \rightarrow Multiplication of all alike ways i.e.,

$$\text{Zero or more} = (n_1 + 1)(n_2 + 1)$$

$$\text{Atleast one} = [(n_1 + 1)(n_2 + 1) - 1]$$

Note: 2 is considered here on the assumption that there is only 2 choices: Select or leave. If there are more choices, increase 2 by such additional choices.

Geometry

○ Straight Line \rightarrow A line can be created by joining any 2 points. Ways = Total Points C_2

○ Triangle \rightarrow A triangle can be formed by joining any 3 points. Ways = Total Points C_3

Note: A triangle cannot be formed if all 3 points are collinear (in straight line). In such a case, remove the ways in which such collinear points can be selected for triangle.

Eg: Form triangle using 15 points where 8 of them are collinear $\Rightarrow {}^{15}C_3 - {}^8C_3$

○ Diagonals \rightarrow While selecting points in a shape to form a line, the sides of the shape also get selected. Hence, Ways = Total Points C_2 (-) no. of sides in the shape

○ Parallelogram \rightarrow To form a llgm, one needs 2 lines on x axis and 2 lines on y axis

$$\text{Ways} = \text{lines on x axis } C_2 \times \text{lines on y axis } C_2$$

CHAPTER 6

Sequence & Series

Ordered Collection of numbers formed by definitive rule is called as **Sequence**
 Eg: 2, 4, 6, 8 n

Sum of elements of a sequence to n terms is called **Series**
 Eg: 2 + 4 + 6 + 8 + + n

Arithmetic Progression

Sequence in which next term is obtained by **adding** a common difference "d"
 Eg: 1, 2, 3, 4, 5,

To find n^{th} term in AP

$$a_n = a + (n-1)d$$

First term common difference

To find Sum of n terms in AP

$$S_n = n(a + l) / 2 \quad \text{OR} \quad S_n = \frac{n}{2} [2a + (n-1)d]$$

n^{th} term

Sum of series (formulas):

- 1st n natural numbers $(1 + 2 + 3 + 4 + \dots + n) = n(n + 1)/2$
- 1st n odd number $(1 + 3 + 5 + 7 + \dots + (2n - 1)) = n^2$
- Squares of 1st n natural number $(1^2 + 2^2 + 3^2 + 4^2 + \dots + n^2) = n(n + 1)(2n + 1)/6$
- Cubes of 1st n natural number $(1^3 + 2^3 + 3^3 + 4^3 + \dots + n^3) = [n(n + 1)/2]^2$

Geometric Progression

Sequence in which next term is obtained by **multiplying** by a constant multiplier "r"
 Eg: 2, 4, 6, 8, 10,

To find constant multiplier

$$r = a_n / a_{(n-1)}$$

To find Sum of n terms in GP

$$S_n = \frac{a(1 - r^n)}{(1 - r)} \quad \text{OR} \quad S_n = \frac{a(r^n - 1)}{(r - 1)}$$

$|r| < 1$ $|r| > 1$

To find n^{th} term in GP

$$a_n = a r^{(n-1)}$$

First term constant multiplier

To find Sum of ∞ terms in GP

$$S_{\infty} = \frac{a}{(1 - r)}$$

Arithmetic Mean

Value in between 2 terms of an AP.

i.e., 'b' is the AM between 'a' and 'c'

in an AP of a, b, c where

$$b = \frac{a + c}{2}$$

Geometric Mean

Value in between 2 terms of an GP.

i.e., 'b' is the GM between 'a' and 'c'

in a GP of a, b, c where

$$b^2 = ac$$

Solving Sequence & Series**Sequence is given. Find****nth formula**

Substitute n = 1, 2, 3 in options. Find option matching sequence

Sum formulaFind S₁, S₂, S₃ from sequence. Substitute n = 1, 2, 3 in options. Find option matching with the sums**Find sequence by using****AP/GP formula**

Substitute n = 1, 2, 3 in formula and arrive at sequence.

Sum formulaSubstitute n = 1, 2, 3 in formula and arrive at S₁, S₂, S₃
a₁ = S₁, a₂ = S₂ - S₁
a₃ = S₃ - S₂

Note: If AP/GP formula is asked and sum formula is requested or vice versa, arrive at sequence using first formula and substitute in second formula to verify.

CHAPTER 7

Sets, Relations & Functions

Sets	collection of well-defined distinct objects	Roster form $A = \{a, e, i, o, u\}$	Set builder form $A = \text{set of vowels}$

Properties of sets

- **Null set:** Empty set containing no elements. Represented by $\{\}$ or ϕ
- **Singleton set:** Set containing only 1 element. Eg: $\{1\}$
- **Equal set:** Set containing only 1 element. Eg: If $A = \text{set of natural numbers}$, $B = \text{set of +ve integers}$, then $A = B$
- **Subsets:** The number of sub-sets of a set is 2^n . If $A = \{1, 2, 3\}$, subset = $2^3 = 8$
 $\phi, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}$
 Subset are denoted by \subset or \subseteq . i.e., $\{1, 2\} \subset \{1, 2, 3\}$
- **Proper Subsets:** Sub-sets does not include the main set. $\Rightarrow 2^n - 1$, i.e., $2^3 - 1 = 7$
 $\phi, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}$
- **Power Set:** Collection of all subset
 $P(A) = \{\phi, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}$

Relation between sets

For the given sets, $A = \{1, 2, 3\}$, $B = \{1, 3, 5, 7, 9\}$ and $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

- $S \Rightarrow S$ is the **universal set** since it contains all elements of A & B
- $A \cap B \Rightarrow A$ **intersection** B contains all **common** elements in both sets i.e., $\{1, 3\}$
- $A \cup B \Rightarrow A$ **union** B contains all **unique** elements in both sets i.e., $\{1, 2, 3, 5, 7, 9\}$
- $A - B \Rightarrow$ Contains all **unique** elements in A which are not present in B i.e., $\{2\}$
- $B - A \Rightarrow$ Contains all **unique** elements in B which are not present in A i.e., $\{5, 7, 9\}$
- $A' \Rightarrow A$ **complement** refers to all elements of S which is not present in A i.e., $\{4, 5, 6, 7, 8, 9, 10\}$

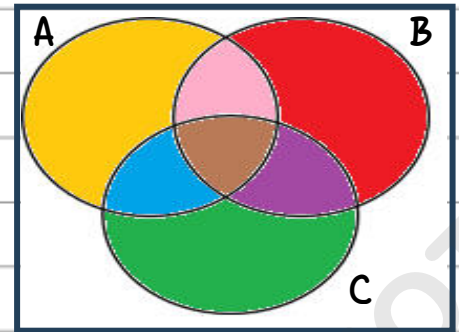
Rules of Set

- $A \cap S = A$
- $A \cup S = S$
- $(A \cup B)' = A' \cap B'$
- $(A \cap B)' = A' \cup B'$

○ $A \times B \Rightarrow$ **Cartesian Product** of A & B refers to all pairs of (a, b) where "a" belong to Set A and "b" belongs to set B i.e., $\{ (1,1), (1,3), (1,5), (1,7), (1,9), (2,1), (2,3), (2,5), (2,7), (2,9), (3,1), (3,3), (3,5), (3,7), (3,9) \}$

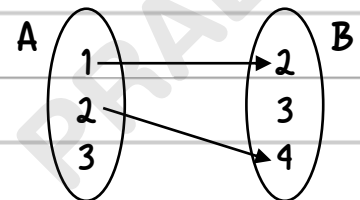
Euler - Venn Diagram

- $n(A \cup B) \Rightarrow n(A) + n(B) - n(A \cap B)$
- $n(A \cup B \cup C) \Rightarrow n(A) + n(B) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$
- $n(\text{Only } B \ \& \ C) \Rightarrow n(B \cap C) - n(A \cap B \cap C)$
- $n(\text{Only } C) \Rightarrow n(C) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$



Relations

instance where **atleast 1 element** of Set A has a mapping to Set B



Types of Relations

Reflexive

If Element a is related to itself
Eg: a is equal to a

Symmetric

If a is related to b, then b is related to a
Eg: If a knows b, then b knows a

Transitive

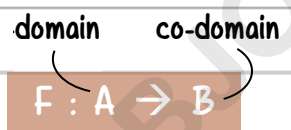
If a is related to b & b is related to c, then a is related to c
Eg: If $a \parallel b$ & $b \parallel c$ then, $a \parallel c$

Equivalence

If a relation is reflexive, symmetric & transitive
Eg: $x = y$

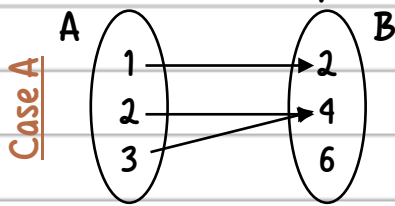
functions

a **relation** where **all elements** of Set A are mapped to **any 1 element** of Set B.



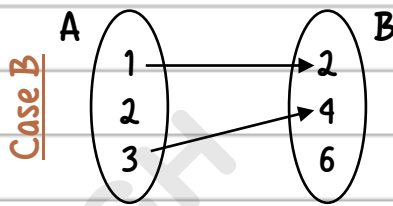
NOTES:

○ It is not necessary for elements in Set A to have a unique element in Set B.



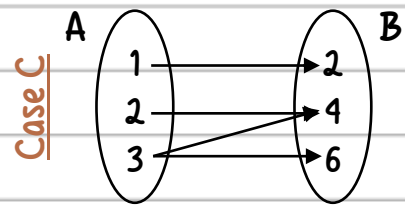
Function

Set A has 1 image each in B



NOT a function.

2 is not mapped



NOT a function.

3 is mapped twice

○ **Domain:** All Elements of Set A

○ **Co-Domain:** All Elements of Set B (whether mapped or not)

○ **Range:** Elements of Set B which are mapped to Set A

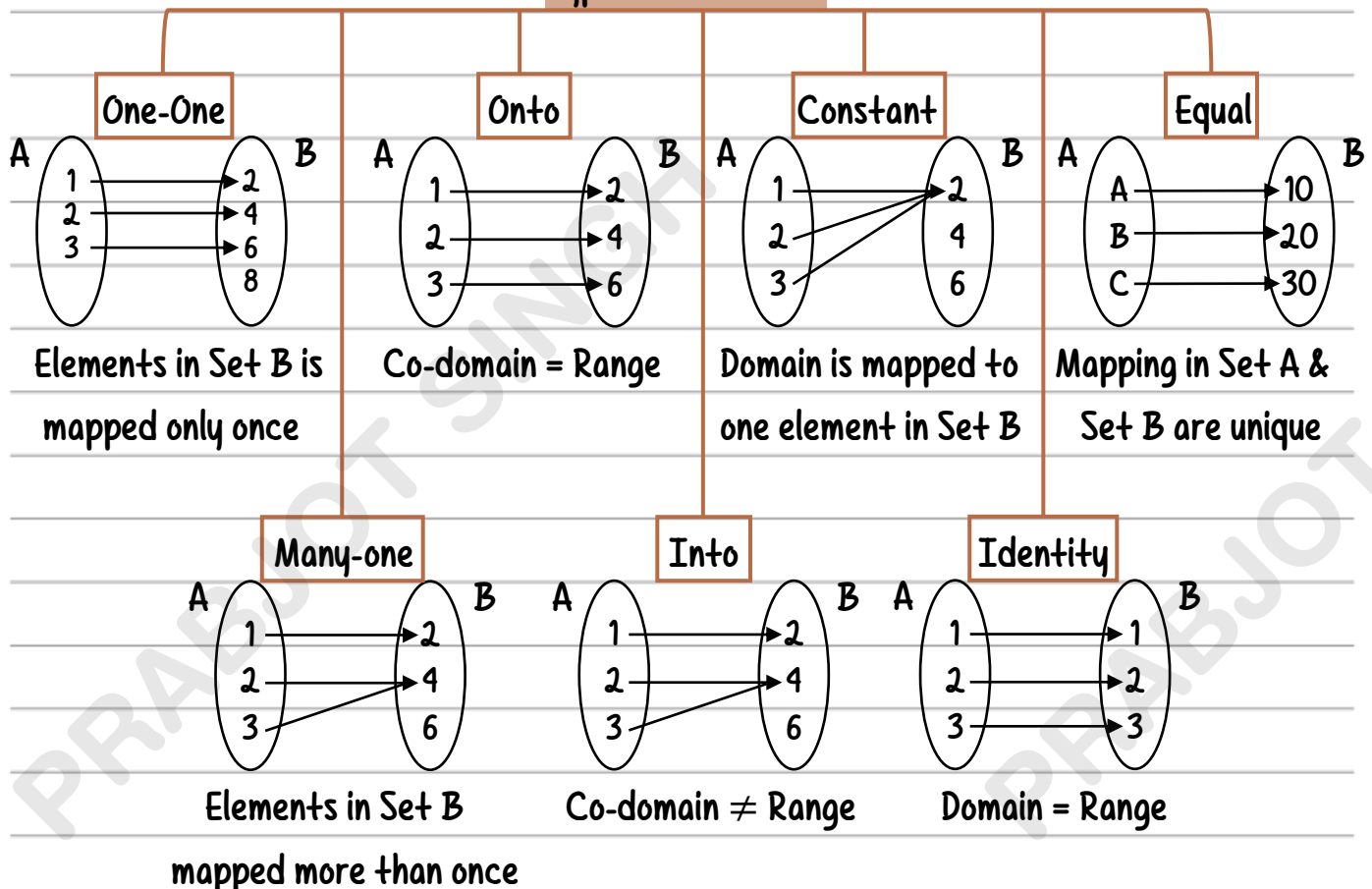
Eg: In Case A, Domain = {1, 2, 3}, Co-domain = {2, 4, 6}, Range = {2, 4}

Note: If the elements are not finite, express them using circle or square bracket

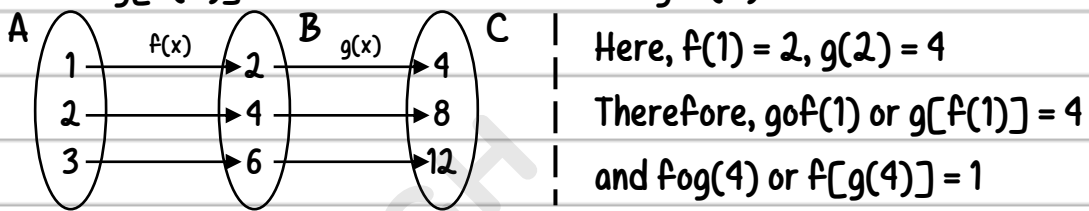
[Refer Linear Inequality Page 7]

○ In **graphical** terms, a relation becomes a function if a parallel line drawn to y axis intersects with the equation at **only one point**.

Types of functions



Composite function Where 2 functions are involved such that $f(A) = B$ and $g(B) = C$, then it can be said that $g[f(A)] = C$. This can be written as $g \circ f(A)$



Inverse function If f is a **one-one** and **Onto** function, such that $f(x) = y$, then inverse function $f^{-1}(y) = x$. For eg: if $f(1) = 2$, then $f^{-1}(2) = 1$.

Steps to find inverse function of $f(x)$ [say $f(x) = 2x$]

Step 1: Write the given equation in terms of y i.e., $y = 2x$

Step 2: Derive the equation of x in terms of y i.e., $x = y/2$

Step 3: Replace x with y and y with x in all places i.e., $y = x/2$

$$\Rightarrow f^{-1} = x/2$$

chapter 7

Limits & Continuity

limit

value that a function approaches the output for the given input values

Steps to solving limits**Step 1:** Determine whether the limit exists.

A limit for a function only exists when Left hand limit = Right hand limit

$$\lim_{x \rightarrow a^-} f(x) = \lim_{x \rightarrow a^+} f(x)$$

Step 2: Substitute "a" in $f(x)$ to check if the value is determinate or undefined

Undefined	Defined
$\frac{a}{0}, \frac{\infty}{\infty}, \infty - \infty, 1^\infty, \sqrt{-a}, 0^\infty, 0^0, \infty^0$	$\frac{0}{a} = 0, \frac{a}{\infty} = 0, \infty + \infty = \infty$

Step 3: If $f(x)$ is determinate, compute value of limit substituting a in $f(x)$ **Step 4:** If value is undefined, using any of the following methods, to make $f(x)$ determinate.

Factorisation method	Differentiate method	Rationalisation method
<i>(applicable in case of fraction with quadratic equations)</i>	<i>(applicable in case value of $f(x)$ is $0/0$ or ∞/∞)</i>	<i>(applicable if $f(x)$ has irrational equations)</i>
Factorise equations to eliminate common factor in Nr and Dr to remove the undefined form	Differentiate Nr & Dr until $f(x)$ becomes determinate	Multiple Nr and Dr with conjugate pairs. Eliminate undefined form
	Note: $\frac{dy}{dx} x^n = n(x)^{n-1}, \frac{dy}{dx} a = 0$	

Calculator trick*(applicable only in case of above four and not when standard functions are used)*If $x \rightarrow a$, substitute the x in the equation with marginally high number, eg: 0 becomes 0.01If $x \rightarrow \infty$, substitute the x in the equation with 100

The nearest option resembling with value arrived in the calculator is the answer

Standard functions of limits:

$$\lim_{x \rightarrow 0} \frac{e^{f(x)} - 1}{f(x)} = 1$$

$$\lim_{x \rightarrow 0} \frac{a^{f(x)} - 1}{f(x)} = \log a$$

$$\lim_{x \rightarrow 0} \frac{\log[1+f(x)]}{f(x)} = 1$$

$$\lim_{x \rightarrow a} \frac{f(x)^n - a^n}{f(x) - a} = n(a)^{n-1}$$

$$\lim_{x \rightarrow \infty} f(x)^x = e^{x[f(x)-1]}$$

continuity

a function which at a given interval has no break in the graph of the function in the entire interval range

A function is said to be continuous only if the following condition is satisfied:

$$\lim_{x \rightarrow a^-} f(x) = \lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a^+} f(x)$$

$$\text{OR } f(a^-) = f(a) = f(a^+)$$

Modulus function

Modulus function (or absolute value function) means reporting only +ve value of the function as the output. For eg, $|2| = 2$ and $|-2|$ is also 2

In case of a modulus function i.e., $|f(x)|$, the limit of the functions can be arrived as follows:

$$f(x) = \begin{cases} f(x) & \text{for } f(x) \geq 0 \\ -f(x) & \text{for } f(x) < 0 \end{cases}$$